MULTIMEDIA UNIVERSITY

STUDENT ID NO

Table No.:

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

TMA1101 – CALCULUS
(All sections / Groups)

16 OCTOBER 2018
2:30 PM – 4: 30 PM
(2 Hours)

For examiner's use.

Question	Marks
1	
2	
3	
4	
5	
Total	

INSTRUCTIONS TO STUDENT

1. This question paper consists of eleven pages with **FIVE** questions.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Write your answers in the question paper itself.
4. **No calculators are allowed**.
5. **You are required to write proper steps.**

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ANSWER ALL QUESTIONS.

QUESTION 1 [10 marks]

(a) Find the following limits. [2 marks]

[***You must show at least one intermediate step where*** $\lim\limits_{x \to c}$ ***is still needed.***]

(i) $\lim\limits_{x \to 2} \dfrac{2x^2 - 3x - 2}{(x+2)(2x-1)}$

(ii) $\lim\limits_{x \to \infty} \dfrac{3x^2 + \cos x}{x^2 + 2}$

Continued ....

1. (b) Given $f(x)=\begin{cases} x+3, & \text{if } x\le 3 \\ x+5, & \text{if } 3<x\le 4 \\ x^2-8, & \text{if } x>4 \end{cases}$ [4.5 marks]

(i) Find $f(4)$.

(ii) Determine $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$.

[***For this part, you must show at least one intermediate step where*** $\lim_{x\to 4^-}$ ***or*** $\lim_{x\to 4^+}$ ***is still needed.***]

(iii) Does $\lim_{x\to 4} f(x)$ exist? Give your reason. If it exists, state its value.

(iv) Is the function f continuous at 4? Give the reason for your answer.

(c) [3.5 marks]

(i) State the Intermediate Value Theorem
(i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $2x^4-5x^3-6=0$ in the interval $[-1,0]$. You must write proper steps to arrive at the conclusion; just writing some calculations would not be enough.

Continued …….

QUESTION 2 [10 marks]

(a) Use the formal definition of derivative to find $f'(2)$ when $f(x) = x^2 + x$.
[***You are reminded to write proper steps.***] [2.5 marks]

(b) Find $\dfrac{dy}{dx}$ with y as given. [3 marks]
[***Use the product rule or the quotient rule for differentiation; show proper steps.***]

(i) $y = e^{2x} \sin 3x$

(ii) $y = \dfrac{e^{-x}}{3 + e^{x}}$

Continued …….

2. (c) The point (2, –1) lies on the curve $y^3 - x^2y + 2x^3 = 19$. [4.5 marks]

Use implicit differentiation to obtain $\frac{dy}{dx}$ **in terms of x and y.**

Then find the equation of the tangent to the curve $y^3 - x^2y + 2x^3 = 19$ at the point (2, –1).

Continued ....

QUESTION 3 [10 marks]

(a) [3.5 marks]

(i) Use $\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$ to find the values of A and B which make the equation $\cos 2\theta \sin 4\theta = A\sin 6\theta + B\sin 2\theta$ an identity.

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \cos 2x \sin\ 4x\, dx$

(b) [3 marks]

(i) Determine the values of A and B in the following partial fraction decomposition.

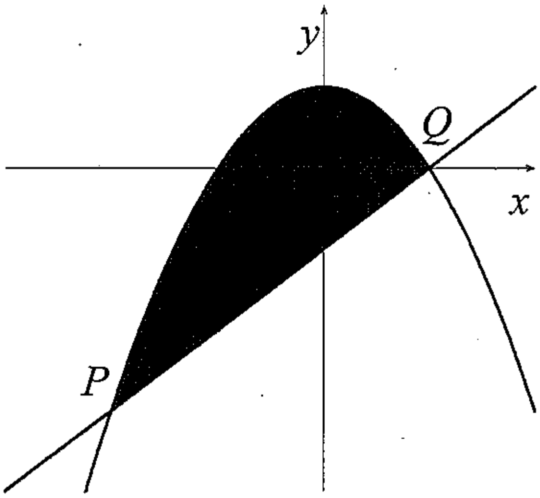
$$\frac{x+34}{x^2-4x-12} = \frac{A}{x-6} + \frac{B}{x+2}$$

(ii) Integrate $\int \frac{x+34}{x^2-4x-12} dx$

Continued ....

3. (c) [3.5 marks]

The figure shows a region bounded by the parabola $y = 1 - x^2$ and the straight line $y = x - 1$.



(i) Determine the x-coordinates of the points of intersection P and Q.

(ii) Write down a definite integral that can be used to find the area of this region and proceed to find the area.

Continued ....

QUESTION 4 [10 marks]

(a) Write down the condition on r for the convergence and divergence of the infinite **geometric series** $\sum_{n=1}^{\infty} ar^{n-1}$. $(a \neq 0)$

Then determine if the geometric series $\sum_{k=1}^{\infty} \frac{5^k}{3^{2k}}$ is convergent or divergent.

[2 marks]

(b) Use the **ratio test** to determine whether the following series is convergent.

$$\sum_{n=1}^{\infty} \frac{e^n}{n^2}$$

[2 marks]

Continued ....

4. (c) Obtain the first few derivatives of $f(x) = \sin 2x$. [3 marks]
Use these to derive the **Maclaurin polynomial** of order 3 for $f(x) = \sin 2x$.

(d) A periodic function $f(x)$ with period 2π is defined as [3 marks]

$$f(x) = \begin{cases} -1, & \text{if} \quad -\pi \le x < -\dfrac{\pi}{2} \\ 0, & \text{if} \quad -\dfrac{\pi}{2} \le x < \dfrac{\pi}{2} \\ 1, & \text{if} \quad \dfrac{\pi}{2} \le x < \pi \end{cases}$$

(i) Sketch a graph of $f(x)$ in the interval $-2\pi < x < 2\pi$

(ii) The **Fourier series** of $f(x)$ has the form $f(x) \sim \dfrac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$.

Determine the value of b_3.

Continued ....

QUESTION 5 [10 marks]

(a) Given $F(x,y) = 5xy^2 + \sin x - e^{xy}$, find the partial derivatives $\dfrac{\partial F}{\partial x}$ and $\dfrac{\partial F}{\partial y}$.

[1 mark]

(b) Solve the first order **separable equation** $\dfrac{dy}{dx} = \dfrac{1}{x^2 y^2}$ subject to the initial condition $y(1) = 1$.

You may leave your answer in implicit form. [2.5 marks]

Continued ....

5. (c) You are told that e^{2x} is an integrating factor for the first order linear equation $\frac{dy}{dx}+2y=4e^{2x}$ subject to the initial condition $y(0)=2$.
Solve the equation and give your solution in explicit form.

[3 marks]

(d) [3.5 marks]

(i) Find the roots of the characteristics equation of the homogeneous differential equation $y''-4y'=0$ (i.e., $\frac{d^2y}{dx^2}-4\frac{dy}{dx}=0$).
Then write down the general solution y_h of this homogeneous equation.

(ii) If $y=Ae^{2x}$ is a particular solution of the second order differential equation $y''-4y'=3e^{2x}$, determine the value of A.

(iii) Hence, write down the general solution for the differential equation $y''-4y'=3e^{2x}$.

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